

THE SUBJECT MATTER PREPARATION FOR (EFFECTIVE) TEACHING OF
MATHEMATICS

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Abstract

The subject matter preparation, that is *what* and *how much* of content knowledge is needed, and defining effective teaching has been a growing interest among mathematics education researchers lately. The challenge of finding the right balance between content and pedagogy has been inspired by the works of Lee Shulman (1986), Deborah Ball and her colleagues (1990 and 2008), Darling-Hammond (2006) and others. In this paper, I give an overview of subject matter preparation and effective teaching in mathematics by defining the two points, identifying and analyzing different perspectives, and describing some frameworks that deal with subject matter preparation and effective teaching.

The Subject Matter Preparation for (Effective) Teaching of Mathematics

A mathematician should never forget that mathematics is too important to frame its instruction to suit more or less the needs of future mathematicians. (Freudenthal, 1973, p. 69)

Educators, policymakers, school administrators, and even teachers are faced with a common dilemma: what makes a teacher (more) effective in educating our children. Research indicates that teacher preparation and knowledge of teaching and learning, subject matter knowledge (SMK), experience, and qualifications measured by the teacher licensure are contributing factors in making teachers more effective (Darling-Hammond, 2006). However, there are two keys that deserve to bear more weight than the others mentioned above, namely (1) teacher knowledge of subject matter or content knowledge and (2) knowledge and skill in how to teach that subject or pedagogical knowledge. In the past, content knowledge and pedagogical knowledge were treated separately. Lee S. Shulman is considered as the most notable person to address this dichotomy. Shulman introduced and popularized the notion of pedagogical content knowledge (PCK) that includes pedagogical knowledge and content knowledge of teachers¹. In this paper, I will try to explore the PCK with a view of mathematics teaching in mind.

There seems to be always a tension between content knowledge and pedagogical knowledge in teacher education programs in the US (Davis & Simmt, 2006). One line of thought is that teachers need to have a solid foundation and understanding of subject matter not only they want to teach but well beyond (Baker, Bressoud, Epp, Ganter, Haver, & Pollatsek, 2004; Even, 1993; Leitzel, 1991). On the other hand, there are others who believe

¹ <http://www.leeshulman.net/domains-pedagogical-content-knowledge.html>

that teachers should focus on the materials they will teach in classroom and stress more on delivering the contents or developing mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008). However, most research put emphasis on streamlining the two approaches (Davis & Simmt, 2006; Grossman, Stodolsky, & Knapp, 2004).

With these lines of thoughts in mind, I structure the paper into three parts. First, I will give an overview and definition of SMK, PCK and what it means by effective teaching. Second, I will explore the importance and perspectives of SMK and teaching by analyzing (1) the importance, (2) the US perspective, and (3) the International perspective of the two. Lastly, I will describe some key frameworks that were developed based on the PCK.

Overview of SMK, PCK and Effective Teaching

The issue of teachers' knowledge of mathematics, especially finding the balance between content knowledge and pedagogical knowledge, has been in forefront for several decades now. Yet, very little progress has been made in finding a consensus among researchers in this very issue. A common approach for teachers' preparation program is to have pre-service teachers to take a set of "stock" courses such as calculus, linear algebra, discrete mathematics, and introductory statistics (Davis & Simmt, 2006). It is regarded as a common sense approach, generally liked by the administrators and policymakers, thinking more mathematics will make them effective teachers. However works of Begle (Begle, 1979) show that there is at best a weak relationship between the courses taken by pre-service teachers and students' performances on standardized and exit exams. This type of results does not promote the common sense approach. On the other hand, scholars like Freudenthal call for more in-depth understandings of topics in conventional curriculum. However, one cannot deny that it takes a solid foundation in mathematics to have sound knowledge of conventional

curriculum topics. The foundation is developed by taking depth and breadth of mathematics courses. So the dilemma continues between more math, in-depth math, and content specific math.

PCK is the term introduced by Shulman (Shulman, 1986) to combine pedagogical knowledge and content knowledge. Pedagogical knowledge addresses the *how* of teaching. It is generally gained through course works in education and experience. Content knowledge, on the other hand, addresses the *what* of teaching. It is gained through conventional mathematical courses as well as teachers own disposition. Shulman proposes three forms of teacher knowledge: (1) propositional knowledge, (2) case knowledge, and (3) strategic knowledge. Propositional knowledge is regarded as the way teachers are taught, as well as the way “we examine the research on teaching and learning and explore its implications for practice” (Shulman, 1986, p. 10). There are three types of propositional knowledge: disciplined empirical or philosophical inquiry, practical experience, and moral or ethical reasoning. Case knowledge is knowledge of specific well-documented and richly described events. The third type of knowledge, the strategic knowledge, comes into play when teachers are confronted with problems that directly “collide with the principles” (Shulman, 1986, p. 13). In general, Shulman’s PCK is a form of practical knowledge that is used by teachers to guide their actions in highly contextualized classroom settings. PCK is also concerned with the representation and formulation of concepts, pedagogical techniques, and knowledge of what makes concepts difficult or easy to learn. So PCK approach requires both content and pedagogical knowledge in training pre-service teachers.

Effective (Mathematics) teaching is a complex issue. Much of the complexity arises from defining what is effective, as well as what type of knowledge and practices make a

teacher effective. As McCrory et al. explains, “Whatever knowledge is best, the link between content knowledge and effective mathematics teaching is not well understood, even though it is logically compelling to argue that it matters” (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012, p. 585). However we can establish some standards for effective teaching.

Effective teachers should

- Understand and be able to apply strategies in helping students to gain knowledge in mathematical concepts
- Identify and use the knowledge children bring into the class and use it to motivate and engage students in mathematical discourses
- Diagnose individual learning needs and provide support as needed
- Develop a positive climate in the classroom in order to make a stimulating learning environment.

Importance and Perspective of SMK and Teaching

Importance of SMK

Teaching and helping students to understand mathematics requires more than delivering facts, formulas, and procedural knowledge. Mathematics is a very powerful tool that can be used to inquire and gain control over every day and real-world problems.

Teachers’ own understanding of mathematics and appropriate pedagogical approach will influence children immensely. Ball and McDiarmid (1990) wrote that “Philosophical arguments as well as common sense support the conviction that teachers’ own subject matter knowledge influences their efforts to help students learn subject matter” (p. 2). If teachers have inaccurate knowledge or view of knowledge, it may get passed to their students. As well as, teachers’ narrow view of knowledge may fail them to identify to students’ line of

thinking and challenge their misconceptions. “Subtly, teachers’ conceptions of knowledge shape their practice – the kinds of questions they ask, the ideas they reinforce, the sorts of tasks they assign” (Darling-Hammond, 2006).

Mathematics teachers need to deeply understand the mathematical ideas that are central to the grade levels they want to teach. Teachers cannot have just the procedural knowledge of the appropriate grade level mathematics. They need to know how to represent and connect mathematical idea so that students may comprehend them and appreciate the power, and diversity of these ideas. Teachers also need to understand students’ thought process to help them understand questions such as:

- Whatever I do in one side of equation I must do the same thing on the other side of equation to keep it balanced. So what is wrong if I add 1 to numerator of both fractions in $\frac{1}{2} = \frac{2}{4}$ and get $\frac{2}{2} = \frac{3}{4}$?
- Why $(-3) \times (-5) = 15$?
- Why should I learn quadratic formula when I can use calculator to find roots to 8 decimal places?

To answer these types of questions teachers need to have mathematics beyond procedural knowledge. Mathematics teachers should not only learn important mathematics, but they should also explicitly see the fundamental connections between what they are learning and what they teach in their own classrooms. A solid foundation through appropriate subject matter knowledge preparation will enable them to make such connections.

US perspective on SMK

As mentioned earlier, the biggest challenge in US teacher educator programs is to find a balance and emphasis on what courses specifically prepare teachers for mathematics teaching, as well as provide professional development for practicing teachers. In general, most teacher educator programs focus either on mathematics content or pedagogy. Very few programs focus on the “nuanced combination” (Even, 1993) of the two. So, more guidance and research is needed about what works for teachers to develop an effective mathematics teaching.

In the US, the elementary teacher track programs have about half of their courses in liberal arts and some courses in education. Recent initiatives in states like New Jersey, California, Illinois, Texas, and Virginia – have drastically reduced or eliminated the number of education courses pre-service teachers need to take (Ball & McDiarmid, 1990). Elementary teachers take several introductory courses in broad range of areas such as: history, English, arts, sociology, psychology, biology etc. and develop an understanding of subject matters at the surface level. On the secondary teacher track programs, students take more content courses on the subject they want to teach and four to five teacher preparation courses in addition to student teaching. Usually, secondary teachers major in the discipline they want to teach. So, content knowledge is emphasized more. However, one might question if this is enough of content courses to give teachers a solid foundation. Researchers like Deborah Ball also emphasize that we ought to look at the prospective teachers school works, as they spend “13 years in school prior to entering college” (Ball & McDiarmid, 1990, p. 6). Additionally, a good amount of teaching disposition is developed in teachers’ in-service years through their experience. So, in the US, there has been a growing interest in prospective teachers’ pre college and post college learning in addition to collegiate preparation.

International perspective on SMK

International approaches in subject matter preparation and developing effective teachings is very divergent and varies across the Continents, Regions, and Countries. Industrialized nations, such as Singapore, Republic of Korea, Belgium, Germany Japan etc., invest heavily in teacher preparation programs and perform well according to TIMSS data². On the other hand, smaller and not so affluent nations, such as Ghana, Morocco, Tunisia, Chile, cannot afford much to spend in their teacher preparation program and appear at the bottom of the TIMSS list³. This is a very broad generalization and students' performance on TIMSS tests is affected by, besides teachers' preparation, other various factors as well. We can get an overview of industrialized nation's teacher preparation program from National Council for Accreditation of Teacher Education report ((NCATE), 2013):

- In France, teacher preparation program requires three years of study in the discipline to be taught, followed by two-year subject matter area (content-pedagogy) study at a teacher training institution. New teachers are paired with senior teacher for two years.
- In Germany, teachers need major in two or more areas and pedagogy for secondary teaching, and major in one subject area and pedagogy for elementary teaching. New teachers need to complete two years of student teaching with reduced class schedule and pass the examination on teaching ability.
- In Japan, teachers need a compulsory year-long induction program after preparation. An induction program includes schools-based mentoring for 90 days, lectures and

² <http://nces.ed.gov/pubs2013/2013009rev.pdf> page 10-12.

³ <http://nces.ed.gov/pubs2013/2013009rev.pdf>, page 10-12

practical training for at least 30 days, and nine-day retreat at regional professional development centers.

There are examples of industrialized nations dealing with same dilemmas as we are facing in the US. The Psychology of the Mathematics Education (PME) addressed some of these dilemmas in their 2004 meetings at Bergen, Norway (Doerr & Wood, 2004). In Brazil, according to Marcelo Borba in PME report (Doerr & Wood, 2004), researchers believe that they need to search for the particulars of mathematics that should be taught to teachers with a broad view of mathematical content, not just adding more content to teacher preparation programs. Additionally, they are interested in how different cultural groups produce different mathematics. In Israel, according to Ruhama Even in PME report (Doerr & Wood, 2004), regular university or college mathematics courses do not support the development of adequate mathematical knowledge for teaching secondary school mathematics. In Taiwan, according to Fou-Lai Lin in PME report (Doerr & Wood, 2004), pre-service teachers are experiencing two constructing views about learning mathematics, one from university mathematics courses and the other from mathematics education courses they take. Other research, focusing on SMK and effective teaching internationally, has identified similar dilemma and challenges for different countries (Adler & Davis, 2011). In light of the examples mentioned, we see that there are differences in teacher preparation programs across countries with respect to subject matter preparation. The differences are both in perspectives and how much the country can invest in teacher preparation programs. However, just like in the US, the dilemma of *what* and *how much* of mathematics should be taught and defining effective teaching in teacher preparation programs still exists.

Frameworks in Developing Teachers Knowledge and Pedagogy

As evident from the discussions above, neither the subject matter knowledge nor the pedagogical knowledge alone addresses the shortcomings and challenges of our mathematics teacher educator preparation programs. Recently researchers are interested developing frameworks that tries to streamline the two lines of thoughts ((CPTM), 2013; Hill, Ball, & Schilling, 2008; Davis & Simmt, 2006; McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012). Here I will explore some frameworks that address the SMK, pedagogical knowledge, and categories and practices for effective mathematics teaching. Since the area is quite broad, I will limit my focus on secondary mathematics teaching only.

Mathematical Understanding for Secondary Teaching (MUST) Framework

The MUST framework was originated “with a desire to characterize mathematical knowledge for teaching at the secondary level” ((CPTM), 2013). The framework was enthused by the work of Deborah Ball and her colleagues at University of Michigan. One key characteristic of MUST framework is that it was developed out of classroom practices and examples were drawn from variety of classroom *situations*⁴. The MUST framework encompasses the practices of prospective teachers, student teachers, and practicing teachers. Thus it could be classified as a bottom up approach. The framework is also one of the few, if not only, that focus solely on mathematical understandings at the secondary school level.

MUST framework incorporates three components: (1) mathematical proficiency, (2) mathematical activity, and (3) mathematical work of teaching. *Mathematical proficiency* refers to conceptual understanding and procedural fluency that teachers need for themselves and seek to foster in their students. The mathematical proficiency of teachers needs to be well beyond the secondary level that covers the mathematics in both elementary and college level.

⁴ Also known as Situations Project.

Mathematical activity is the work of “doing mathematics” ((CPTM), 2013, chapter 2, p. 3).

The framework proposes a more conscious and elaborated command of activities the teachers employ and they want their students to learn in mathematics classroom. *Mathematical work of teaching* is an understanding of the mathematical thinking of students, particularly the nature of students’ errors and misconceptions. In the process, teachers may be interested in learning students’ prior knowledge, as well as providing foundation and understandings for the mathematics they will be learning later.

The MUST framework is a joint collaboration between mathematics educators at the University of Georgia and the Pennsylvania State University and supported by the National Science Foundation. The researchers have created and refined a set of situations that are relevant to secondary level mathematics classroom. The framework is an important step towards helping pre-service teachers in gaining mathematical understandings they need, as well as aiding them with examples they are likely to encounter in their service years.

Knowledge of Contents and Students (KCS)

The KCS framework is a step towards to conceptualize, identify, measure, and ultimately improve teachers’ PCK defined by Schulman (Hill, Ball, & Schilling, 2008). The framework also addresses the teachers’ ability to design effective instruction and measuring teachers’ skills in motivating students to learn mathematics. This means that teachers must be able to anticipate students’ difficulties and obstacles, hear and respond appropriately to students’ thinking, and choose appropriate examples and representations while teaching. Both in planning and teaching, teachers must show awareness of students’ conceptions and misconceptions about a mathematics topic. The KCS is defined as (Hill, Ball, & Schilling, 2008)

We propose to define KCS as content knowledge intertwined with knowledge of how students think about, know, or learn this particular content. KCS is used in tasks of teaching that involve attending to both the specific content and something particular about learners. (p. 375)

The definition is based on both theoretical and empirical work on teacher knowledge.

KCS incorporates both subject matter knowledge and PCK by using the mathematical knowledge for teaching (MKT) model. However, authors argue that KCS is distinct from

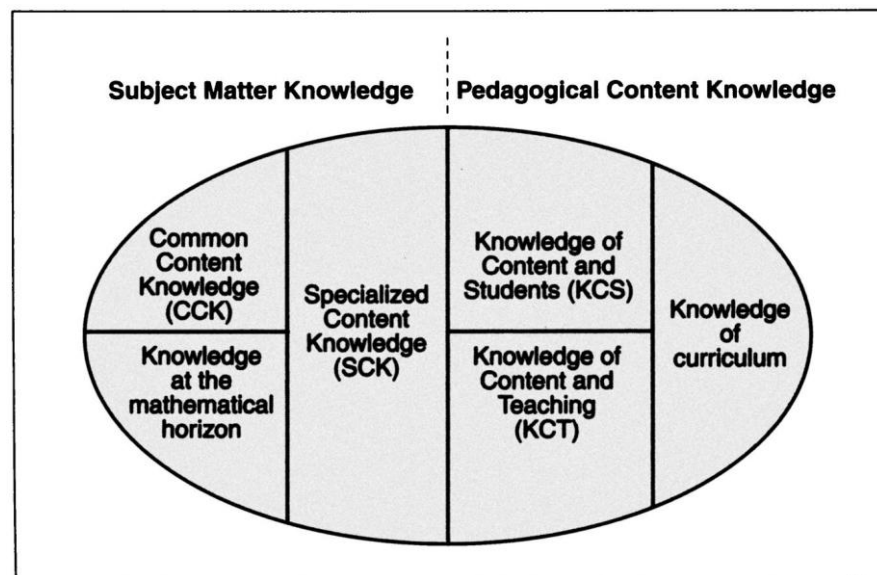


Figure 1. Mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008, p. 377)

teachers' subject matter knowledge. According to KCS framework, a teacher might have strong content knowledge but weak knowledge of how students learn the content or vice versa. KCS provides knowledge of how to: a) anticipate what students are likely to think and b) relate mathematical ideas to developmentally appropriate language used by children. Using the results from multiple choice items, Hill, Ball, and Schilling (2008), reports that this domain remains under-conceptualized and understudied, and further investigation is needed (p.397).

Mathematics for teaching and Knowledge of Algebra for Teaching (KAT) Framework

Mathematics educators Brent Davis and Elaine Simmt (2006) applied the term *mathematics-for-teaching*, using mathematics in terms of complex dynamics, to argue that knowledge of established mathematics is inseparable from knowledge of how mathematics is established. In their work, Davis and Simmt illustrate four intertwining categories of teachers' *mathematics-for-teaching* by investigating the topic of multiplication.

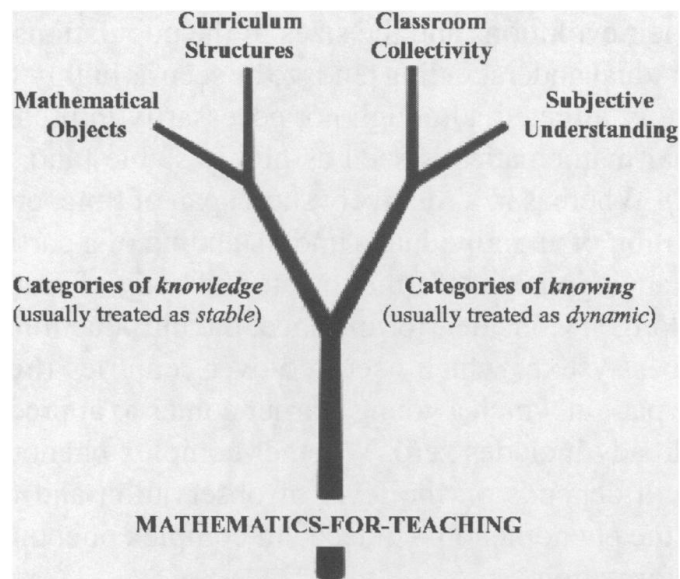


Figure 2. Perceived relationships among some aspects of teachers' mathematics-for-teaching
(Davis & Simmt, 2006, p. 298)

The categories are: (1) Subjective Understanding, (2) Classroom Collectivity, (3) Curriculum Structures, and (4) Mathematical Objects. The first two are regarded as categories of knowing, thus treated as dynamic. The last two are categories of knowledge and treated as stable. Davis and Simmt further argue that it takes seconds for dynamic knowledge to transform, whereas it takes decades for the static knowledge. They stress that a strong sense of these dynamics are critical to effective pedagogy and a core aspect of their mathematical knowledge (Davis & Simmt, 2006, p. 297). Through their work in *mathematics-for-teaching*,

Davis and Simmt propose that (1) some fluency is needed with the four aspects proposed for mathematics teaching, and (2) in mathematics intended for teachers, these aspects can guide towards appropriate emphasis for developing courses.

Joan Ferrini-Mundy and her colleagues at Michigan State University introduced the term *Knowledge of Algebra for Teaching (KAT)* in proposing a two dimensional framework, involving teaching and categories of knowledge, to conceptualize the knowledge for (algebra) teaching (McCrorry, Floden, Ferrini-Mundy, Reckase, & Senk, 2012; Ferrini-Mundy, Floden, & McCrorry⁵). The framework measures both the teachers advanced mathematical knowledge and knowledge closer to teaching by analyzing three categories of teaching practices: (1) decompressing, (2) trimming, and (3) bridging. Decompressing is attaching fundamental meaning to symbols/variables/algorithms that we use in an automatic and unconscious way. In other words, emphasizing more on fundamental knowledge than procedural knowledge. Trimming, as the name implies, involves trimming of mathematical or contextual content while retaining important mathematical features. Finally, bridging involves making connection of students' understandings to the goals of the teacher is seeking to meet, relating the ideas of school algebra to those of abstract algebra, linking one area of school mathematics to another. KAT framework was also used to analyze the instructional materials (Senk & Thompson, 2006), as well as other qualitative and quantitative studies.

Conclusion

The subject matter knowledge with respect to effective teaching is still a new concept. Not too long ago, when subject matter knowledge, especially for elementary and secondary

⁵ <http://www.educ.msu.edu/kat/matrix.htm>

school level, was taken for granted or treated as “common sense” (Ball & McDiarmid, 1990). But works of Ball et al. (1990 and 2008), Davis & Simmt (2006), and Evan (1993) show that teachers have gaps and misconceptions on their knowledge just like their students. We have to realize that teachers today are coming from the same elementary and secondary schoolings where they rarely developed deep understanding of subject matter knowledge. This might explain the reason behind teachers inadequate SMK and students’ low achievement levels. A teacher develops effective mathematics teaching strategies through his/her experience in both in and outside classroom with help of solid SMK foundation. Thus we need to look at how preservice teachers can increase their knowledge of subject matter they want to teach, the experiences that are helpful in developing effective teaching strategies, and continue research on frameworks that help them in effective subject matter preparation. Research, in general, supports that teachers need to have better subject matter preparation to be an effective teacher (Even, 1993). However, that does not mean to have teachers take more content courses. Instead, a balance between subject matter knowledge and pedagogical knowledge is more desirable.

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